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On electroweak symmetry breaking in the littlest Higgs model

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Abstract. In $SU(5)/SO(5)$ little Higgs models radiative corrections give rise to $SU(2)_L \times U(1)_Y$ symmetry breaking. In this work we start a program for a detailed determination of the relevant terms of the effective Higgs potential by computing the contribution of the t, b and T quarks at the one-loop level, as a starting point for a higher-loop computation. In spite of the fact that some two-loop level contributions are well known to be important, we use our preliminary one-loop result to illustrate that, by demanding the effective potential to reproduce exactly the standard model Higgs potential, and in particular the relation $m_H^2 = 2\lambda v^2 = 2\mu^2$, it will be possible to set new constraints on the parameter space of the littlest Higgs model when the computation of all the relevant contributions to the effective Higgs potential is completed.

1 Introduction

In recent years a lot of work has been devoted to the so-called little Higgs models (see [1, 2] for recent reviews). These models use an old suggestion by Georgi and Pais [3, 4], in which the Higgs boson is assumed to be a (pseudo-) Goldstone boson associated to some global spontaneous symmetry breaking [5–10]. For instance, in the case of the paradigmatic littlest Higgs (LH) model [11], $SU(5)$ to $SO(5)$ breaking is assumed to happen at some scale f. The Higgs field is just one of the corresponding 14 Goldstone bosons, and therefore it is, in principle, massless. The $SU(5)$ subgroup $(SU(2) \times U(1))_1 \times (SU(2) \times U(1))_2$ is gauged so that the axial $(SU(2) \times U(1))_{1-2}$ is spontaneously broken, with the gauge bosons typically being heavy (W'^a and B'). The diagonal $(SU(2) \times U(1))_{1+2} = SU(2)_L \times U(1)_Y$ remains unbroken and corresponds to the electroweak SM group. However, radiative corrections coming from the fermionic sector of the model, mainly the third quark generation and an additional vector-like quark T, give rise to an effective potential that produces further spontaneous symmetry breaking of the SM $SU(2)_L \times U(1)_Y$ down to $U(1)_{em}$. In this way some of the Goldstone bosons acquire masses quadratic in the cut-off Λ, which is expected to be of the order of $4\pi f$ [12–14]. The SM model Higgs boson gets a mass that grows only as the logarithm of Λ at the one-loop level. This is the explanation in this setting of why the Higgs boson is expected to be relatively light ($115 \, \text{GeV} < m_H < 200 \, \text{GeV}$). The rest of the Goldstone bosons give masses to the different gauge

bosons through the Higgs mechanism in $SU(5)$ to $SO(5)$ or in SM symmetry breaking. Thus, the LH model explains in a natural and elegant way the expected low value of the Higgs boson mass. In addition this model provides a very rich phenomenology, which could be probed in the next-generation colliders such as the LHC [15, 16]. Since the original proposal of the LH, many other little Higgs versions have appeared [17–21]. Some of these models try to improve the consistency and to reduce the need for fine-tuning in this kind of models [22]. Specially interesting from the phenomenological point of view is the LH version in which the $SU(5)$ gauged subgroup is just $SU(2)_1 \times SU(2)_2 \times U(1)$. In this case, after the first spontaneous broken symmetry, we have only three massive gauge bosons associated to the $SU(2)_{1-2}$ group, W'^a, and four massless gauge bosons, i.e. the SM gauge bosons [16].

Nevertheless, it is clear that any viable little Higgs model has to fulfill the basic requirement of reproducing the SM model at low energies. This implies, in particular, that one should not only have the proper low energy degrees of freedom, but also one should be able to reproduce the action of the SM model as the low energy effective action of the LH model. In this work we compute the contribution of the t, b and T quarks to the effective potential for the SM Higgs doublet $H = (H^0, H^+)$, which gives rise to electroweak symmetry breaking in the LH model. The first terms of this potential are found to have the standard form:

$$V_{\text{eff}} = -\mu^2 H H^\dagger + \lambda (H H^\dagger)^2 \,, \tag{1}$$

with positive μ^2 and λ. Other relevant contributions coming from gauge bosons, scalars and other higher loops are

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expected to go in the opposite direction, but they are also supposed to have a smaller absolute value, so that they do not change the μ^2 and λ signs. The μ^2 sign and value are well known [11, 16], and effectively they are the right ones to produce the electroweak symmetry breaking, giving a Higgs mass $m_H^2 = 2\mu^2$. However, the full expression for λ has not been analyzed in detail. Several relations for the threshold corrections to this parameter in the presence of a 10 TeV cut-off, depending of the UV-completion of the theory, have been reported before [23]. The radiative contributions to λ, at the one-loop level, have not been computed so far.

The computation of the parameter λ is important for several reasons. First, it must be positive for the low energy effective action to make sense (otherwise the theory would not have any vacuum). In addition, from the effective potential above, one gets the simple formula $m_H^2 = 2\lambda v^2$, or, equivalently, $\mu^2 = \lambda v^2$, where v is the SM vacuum expectation value $(H = (0, v)/\sqrt{2})$, which is experimentally set to be $v \simeq 245$ GeV (for instance by the muon lifetime). By computing the effective action for the Higgs doublet in the context of the LH model and taking into account the t, b and T quarks only, i.e. the modes responsible for the electroweak symmetry breaking, it is possible to obtain μ^2 and λ in terms of the λ_T, f and Λ parameters of the LH model, where λ_T is the Yukawa coupling of T, f is the scale of the SU(5)/SO(5) symmetry breaking, and Λ is the ultraviolet cut-off (in fact λ has also a small dependence on the infrared cut-off $m \sim v$). In other words, we find the functions

$$\mu^2 = \mu^2(\lambda_T, f, \Lambda) + \dots,$$
$$\lambda = \lambda(\lambda_T, f, \Lambda) + \dots, \quad (2)$$

where the ellipses include corrections coming from the gauge bosons, scalars and other higher order loops. As is well known, μ depends on the logarithm of Λ at the one-loop level, but λ has also a much stronger quadratic dependence on this cut-off. Moreover, according to the previous discussion, the consistency of the low energy theory sets the following highly non-trivial constraint on the LH model parameters:

$$\mu^2(\lambda_T, f, \Lambda) = \lambda(\lambda_T, f, \Lambda)v^2 + \dots \quad (3)$$

In this work we compute the contribution to these functions coming from the t, b and T quarks present in the LH model at the one-loop level, which are the relevant ones for having symmetry breaking. Then we use this result to illustrate the kind of bounds and restrictions that must be set on the LH model fermion parameters in order to obtain the SM potential from the effective Higgs potential that should also include gauge, scalar and other higher loop contributions [43].

The outline of the paper is as follows: in Sect. 2 we review briefly the LH model and settle the notation we are going to use. In Sect. 3 we compute the t, b and T quark contribution to the effective action for a constant SM Higgs doublet (i.e. the effective potential) by using standard techniques [24], and we obtain the functions $\mu^2 = \mu^2(\lambda_T, f, \Lambda)$ and $\lambda = \lambda(\lambda_T, f, \Lambda)$. Section 4

is devoted to the study of the above-mentioned constraints, and as an example we use our previous result to show the kind of restrictions that it is possible to set on the LH model parameter space from the complete Higgs effective potential. Finally, in Sect. 5 we summarize our main results and present some conclusions and remarks.

2 The model

The LH model is based on the assumption that there is a physical system with a global $G = $ SU(5) symmetry that is spontaneously broken to a $H = $ SO(5) symmetry at a high scale Λ through a vacuum expectation value of order f. Thus the spectrum of the theory will in principle contain 14 Goldstone bosons, including the SM complex doublet $H = (H^0, H^+)$. In addition, the SU(5) subgroup $(\text{SU}(2) \times \text{U}(1))_1 \times (\text{SU}(2) \times \text{U}(1))_2$ is gauged, its diagonal subgroup $(\text{SU}(2) \times \text{U}(1))_{1+2}$ being the SM electroweak group $\text{SU}(2)_\text{L} \times \text{U}(1)_Y$. This group remains unbroken after the SU(5) breaking to SO(5) and consequently the electroweak gauge bosons W_μ^a and B_μ are massless at this level. However, the $(\text{SU}(2) \times \text{U}(1))_{1-2}$ group is spontaneously broken and the corresponding gauge bosons $W_\mu^{'a}$ and B_μ' get masses of order f through the Higgs mechanism. Each of these two gauge groups must commute with a different subgroup SU(3) that acts non-linearly on the Higgs, i.e. when both weak gauge interactions are included, the Higgs boson is a pseudo-Goldstone boson whose mass is protected by the underlying symmetry, but, conversely, if just one of these interactions is considered, the SU(3) symmetry is recovered [11].

With the global SU(5) symmetry breaking into its subgroup SO(5), we have 14 Goldstone bosons, which transform under the electroweak group as a real singlet, a real triplet, a complex doublet and a complex triplet. The real singlet and the real triplet become the longitudinal parts of the B_μ' and $W_\mu^{'a}$ bosons through the Higgs mechanism, and the latter two Goldstone boson multiplets can be interpreted as the SM Higgs doublet and an additional complex triplet; i.e., we still have 10 massless Goldstone bosons. These particles will get radiative masses after the introduction of appropriate gauge and Yukawa couplings to the third-generation t and b quarks and an additional vector-like quark T, the Yukawa contributions being responsible for the expected sign of the μ^2 parameter. Then the magic of the model produces a Higgs mass that is quadratically divergence-free at the one-loop level. Thus we obtain a light Higgs boson in a natural way. The complex triplet is not protected in the same way, and radiative quark corrections make it typically much more massive, thus evading the experimental constraints.

According to the previous discussion, the low energy dynamics of the LH model can be described by a $(\text{SU}(2) \times \text{U}(1))_1 \times (\text{SU}(2) \times \text{U}(1))_2$ gauged non-linear sigma model based on the coset $K = G/H = \text{SU}(5)/\text{SO}(5)$ [24]. The Goldstone boson fields can be arranged in a 5×5 matrix Σ

given by

$$\Sigma = e^{2i\Pi/f}\Sigma_0, \tag{4}$$

where

$$\Sigma_0 = \begin{pmatrix} 0 & 0 & \mathbf{1} \\ 0 & 1 & 0 \\ \mathbf{1} & 0 & 0 \end{pmatrix} \tag{5}$$

has the proper SU(5) symmetry breaking structure, $\mathbf{1}$ being the 2×2 unit matrix, and

$$\Pi = \begin{pmatrix} \xi & \frac{-i}{\sqrt{2}}H^\dagger & \phi^\dagger \\ \frac{i}{\sqrt{2}}H & 0 & \frac{-i}{\sqrt{2}}H^* \\ \phi & \frac{i}{\sqrt{2}}H^T & \xi^T \end{pmatrix} + \frac{\eta}{\sqrt{20}} \, \mathrm{diag}\,(1,1,-4,1,1) \tag{6}$$

is the Goldstone bosons matrix, with $H = (H^0, H^+)$ being the SM Higgs doublet, η being a real scalar, and ξ and ϕ encoding the real triplet and the complex triplet, respectively:

$$\xi = \begin{pmatrix} \frac{1}{2}\xi^0 & \frac{1}{\sqrt{2}}\xi^+ \\ \frac{1}{\sqrt{2}}\xi^- & -\frac{1}{2}\xi^0 \end{pmatrix}$$

and

$$\phi = \begin{pmatrix} \phi^0 & \frac{1}{\sqrt{2}}\phi^+ \\ \frac{1}{\sqrt{2}}\phi^+ & \frac{1}{\sqrt{2}}\phi^{++} \end{pmatrix}. \tag{7}$$

The Lagrangian of the gauged non-linear sigma model is given by

$$\mathcal{L}_0 = \frac{f^2}{8}\mathrm{tr}D_\mu\Sigma(D^\mu\Sigma)^\dagger, \tag{8}$$

where the covariant derivative is defined by [11]

$$D_\mu\Sigma = \partial_\mu\Sigma - i\sum_{j=1}^{2}g_j W_j^a \left(Q_j^a\Sigma + \Sigma Q_j^{aT}\right)$$
$$- i\sum_{j=1}^{2}g'_j B_j \left(Y_j\Sigma + \Sigma Y_j^T\right), \tag{9}$$

where g_j and g'_j are the gauge couplings, $Q_{1ij}^a = \sigma_{ij}^a/2$ for $i,j = 1,2$, $Q_{2ij}^a = \sigma_{ij}^{a*}/2$ for $i,j = 4,5$, and zero otherwise, $Y_1 = \mathrm{diag}(-3,-3,2,2,2)/10$ and $Y_2 = \mathrm{diag}(-2,-2,-2, 3,3)/10$. By diagonalizing the gauge boson mass matrix contained in this Lagrangian one gets the massless W and B SM bosons and the massive W' and B' gauge bosons mentioned above as follows:

$$W^a = c_\psi W_1^a + s_\psi W_2^a,$$
$$W'^a = s_\psi W_1^a - c_\psi W_2^a, \tag{10}$$

where

$$s_\psi = \sin\psi = \frac{g_1}{\sqrt{g_1^2 + g_2^2}},$$
$$c_\psi = \cos\psi = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}, \tag{11}$$

with $M_W = 0$ and $M_{W'} = f\sqrt{g_1^2 + g_2^2}/2$. In a similar way, we have

$$B = c'_\psi B_1 + s'_\psi B_2, \tag{12}$$
$$B' = s'_\psi B_1 - c'_\psi B_2,$$

where

$$s'_\psi = \sin\psi' = \frac{g'_1}{\sqrt{g'^2_1 + g'^2_2}}, \tag{13}$$

$$c'_\psi = \cos\psi' = \frac{g'_2}{\sqrt{g'^2_1 + g'^2_2}},$$

where $M_B = 0$ and $M_{B'} = f\sqrt{g'^2_1 + g'^2_2}/\sqrt{20}$.

A version of the LH models modified such that the gauge subgroup of SU(5) is $[\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y]$ rather than $[\mathrm{SU}(2) \times \mathrm{U}(1)_Y]^2$ has also been introduced [16]. In this case, the covariant derivative is defined by

$$D_\mu\Sigma = \partial_\mu\Sigma - i\sum_{j=1}^{2}g_j W_j^a (Q_j^a\Sigma + \Sigma Q_j^{aT})$$
$$- ig'B(Y\Sigma + \Sigma Y^T), \tag{14}$$

where the generators Q_j^a are the same as in the previous case, and $Y = \frac{1}{2}\mathrm{diag}(-1,-1,0,1,1)$. The field content of the matrix Π in Σ is the same as in the LH model but now there is no B'. This model will be considered in Sect. 4, where some phenomenological consequences of considering the gauge sector are discussed.

Then, at the tree level, the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ SM gauge group remains unbroken. Spontaneous symmetry breaking of this group is produced radiatively mainly by the quark loops from the third generation, which will be initially denoted by u and t; the additional vector-like quark will be denoted by U. The interactions between these fermions and the Goldstone bosons are given by the Yukawa Lagrangian:

$$L_{\mathrm{Yuk}} = -\frac{\lambda_1}{2}f\overline{u}_R\epsilon_{mn}\epsilon_{ijk}\Sigma_{im}\Sigma_{jn}\chi_{Lk} - \lambda_2 f\overline{U}_R U_L + \mathrm{h.c.}, \tag{15}$$

where $m,n = 4,5$, $i,j = 1,2,3$, and

$$\overline{u}_R = c\overline{t}_R + s\overline{T}_R,$$
$$\overline{U}_R = -s\overline{t}_R + c\overline{T}_R, \tag{16}$$

with

$$c = \cos\theta = \frac{\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}},$$
$$s = \sin\theta = \frac{\lambda_1}{\sqrt{\lambda_1^2 + \lambda_2^2}}, \tag{17}$$

and

$$\chi_L = \begin{pmatrix} u \\ b \\ U \end{pmatrix}_L = \begin{pmatrix} t \\ b \\ T \end{pmatrix}_L. \tag{18}$$

Here t, b and T are the mass eigenvectors coming from the mass matrix included in the Yukawa Lagrangian with eigenvalues $m_t = m_b = 0$ and $m_T = f\sqrt{\lambda_1^2 + \lambda_2^2}$. Thus the quark t is massless, and it acquires mass only when the electroweak symmetry is broken, contrary to the quark T, which is massive already at this level.

Next, the Yukawa Lagrangian can be written

$$L_{\text{Yuk}} = \overline{\chi}_{\text{R}} \hat{I}_{3\times3} \chi_{\text{L}} + \text{h.c.}\,, \tag{19}$$

$\hat{I}_{3\times3}$ being the interaction matrix defined below, and

$$\chi_{\text{R}} = \begin{pmatrix} t \\ b \\ T \end{pmatrix}_{\text{R}}.$$

Since we are interested in the computation of the fermion contribution to the SM Higgs effective potential, we set $\xi = \phi = \eta = 0$, and thus the interaction matrix $\hat{I}$ is given by

$$\hat{I} = \begin{pmatrix} -\sqrt{2}\lambda_1 c H^0 \Theta & -\sqrt{2}\lambda_1 c H^+ \Theta & \lambda_1 c \frac{HH^\dagger}{f} \Theta' \\ 0 & 0 & 0 \\ -\sqrt{2}\lambda_1 s H^0 \Theta & -\sqrt{2}\lambda_1 s H^+ \Theta & \lambda_1 s \frac{HH^\dagger}{f} \Theta' \end{pmatrix}, \tag{20}$$

where Θ and Θ' are functions of $HH^\dagger/f^2$ whose expansion starts as follows:

$$\Theta\left(\frac{HH^\dagger}{f^2}\right) = 1 - \frac{2HH^\dagger}{3f^2} + \dots$$
$$\Theta'\left(\frac{HH^\dagger}{f^2}\right) = 1 - \frac{HH^\dagger}{3f^2} + \dots \tag{21}$$

Therefore the complete Lagrangian for the quarks is

$$L_\chi = L_0 + L_{\text{Yuk}} = \overline{\chi}_{\text{R}}(\mathrm{i}\partial\!\!\!/ - M + \hat{I})\chi_{\text{L}} + \text{h.c.}\,, \tag{22}$$

with $M = \text{diag}(0, 0, m_T)$.

In the LH model electroweak symmetry breaking is produced mainly by the three quarks included in the above Lagrangian, whilst the gauge bosons and the complex triplet tend to restore the symmetry. In the following we will consider only the effect of the quarks on the effective Higgs potential.

3 The effective Higgs action and potential

In order to compute the leading fermion contribution to the effective Higgs potential we will start from the effective Higgs action obtained from the t, b and T quarks at the one-loop level, which is an exact computation in this case since the action is quadratic in these fields. Thus, this effective action is given by

$$\mathrm{e}^{\mathrm{i} S_{\text{eff}}[H]} = \int [\mathrm{d}\chi][\mathrm{d}\overline{\chi}] \mathrm{e}^{\mathrm{i} S[H,\chi,\overline{\chi}]}\,, \tag{23}$$

with

$$S[H, \chi, \overline{\chi}] = \int \mathrm{d}^4 x (\partial_\mu H \partial^\mu H^\dagger + L_\chi)\,. \tag{24}$$

By using standard techniques [24], we obtain the following result for the effective action:

$$S_{\text{eff}}[H] = \int \mathrm{d}^4 x \partial_\mu H \partial^\mu H^\dagger + \Delta S\,, \tag{25}$$

with

$$\Delta S = -\mathrm{i}\text{Tr} \log(\mathrm{i}\partial\!\!\!/ - M + \hat{I}) = -\mathrm{i}\text{Tr}(1 + G\hat{I})\,, \tag{26}$$

where we have neglected a constant; it is irrelevant for the computation of the effective action. The operator $G = (\mathrm{i}\partial\!\!\!/ - M)^{-1}$ is just the propagator for the free quarks, which is given by

$$G^{ab}(x, y) \equiv \int \mathrm{d}\widetilde{k} \mathrm{e}^{-\mathrm{i}p(x-y)} (\not{p} - M)_{ab}^{-1}\,. \tag{27}$$

Here $\mathrm{d}\widetilde{k} \equiv \mathrm{d}^4 k/(2\pi)^4$. By expanding the logarithm, the effective action can be written as

$$\Delta S = -\mathrm{i}\text{Tr}\Sigma_{k=1}^\infty \frac{(-1)^{k+1}}{k}(G\hat{I})^k = \Sigma_{k=1}^\infty \Delta S^{(k)}\,. \tag{28}$$

Now in order to obtain the effective potential we only have to consider constant Higgs fields, i.e. we set $\partial_\mu H = 0$. Thus we have

$$\Delta S|_{H=\text{const.}} = -\int \mathrm{d}^4 x V_{\text{eff}}(H)\,. \tag{29}$$

In the following we will take H as a constant. The effective potential can be computed as a power series in $HH^\dagger/f^2$, with arbitrarily higher powers of this parameter. However, in order to produce electroweak symmetry breaking, it is sufficient to compute just the first two terms of this expansion. Thus the effective potential can be written as in (1).

It is then not difficult to see that the computation of the μ^2 and λ parameters requires ΔS to be considered up to the fourth term. The generic one-loop diagrams that must be computed are shown in Fig. 1. By using well known methods it is straightforward to obtain the different contributions after some work. The first one $(k = 1)$ corresponds to the first two diagrams in Fig. 1, and it is given by

$$\begin{aligned} \Delta S^{(1)}[H] &= -\mathrm{i}\text{Tr}(G\hat{I}) \\ &= -\frac{4}{3}\lambda_1 m_T f s \int \mathrm{d}^4 x \left(\frac{HH^\dagger}{f^2} - \frac{(HH^\dagger)^2}{f^4}\right) \\ &\quad \times I_0\left(m_T^2\right)\,, \end{aligned} \tag{30}$$

where the divergent integral $I_0\left(m_T^2\right)$ is

$$I_0\left(m_T^2\right) = \int \mathrm{d}\tilde{p} \frac{\mathrm{i}}{p^2 - m_T^2}\,. \tag{31}$$

In order to make sense of this integral we will use an ultraviolet cut-off Λ, where our effective description of the low energy dynamics breaks down. The result is

$$I_0\left(m_T^2\right) = \frac{1}{(4\pi)^2}\left[\Lambda^2 - m_T^2 \log\left(\frac{\Lambda^2}{m_T^2} + 1\right)\right]\,. \tag{32}$$
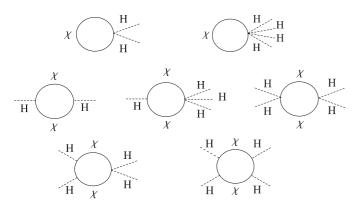
Fig. 1. One-loop diagrams. $H = (H^0, H^+)$ and $\chi \equiv t, b, T$, with all possible combinations of these particles in the loop diagrams

For $k = 2$ (see the three generic diagrams on the second line of Fig. 1), one gets

$$\Delta S^{(2)}[H]$$
$$= \frac{i}{2}\mathrm{Tr}(G\hat{I})^2 = 4\lambda_1^2 \int \mathrm{d}^4x HH^\dagger \left(c^2 I_0(0) + s^2 I_0\left(m_T^2\right)\right)$$
$$+ 2\lambda_1^2 \int \mathrm{d}^4x \frac{(HH^\dagger)^2}{f^4}\left(I_0(0) + 2m_T^2 s^2 I_1\left(m_T^2\right)\right)$$
$$- \frac{16}{3}\lambda_1^2 \int \mathrm{d}^4x \frac{(HH^\dagger)^2}{f^4}\left(c^2 I_0(0) + s^2 I_0\left(m_T^2\right)\right) , \quad (33)$$

where the new divergent integral $I_1\left(m_T^2\right)$ properly regularized is given by

$$I_1\left(m_T^2\right) = \int \mathrm{d}\tilde{p}\frac{i}{(p^2 - m_T^2)^2} = -\frac{1}{(4\pi)^2}$$
$$\times \left[\log\left(\frac{\Lambda^2}{m_T^2} + 1\right) - \frac{1}{1 + \frac{m_T^2}{\Lambda^2}}\right] . \quad (34)$$

The $k = 3$ contribution (see the first diagram in the bottom line of Fig. 1) is

$$\Delta S^{(3)}[H] = -\frac{i}{3}\mathrm{Tr}(G\hat{I})^3$$
$$= -8m_T\lambda_1^3 s \int \mathrm{d}^4x \frac{(HH^\dagger)^2}{f}$$
$$\times \left(c^2 I_2\left(m_T^2\right) + s^2 I_1\left(m_T^2\right)\right), \quad (35)$$

where the divergent integral $I_2\left(m_T^2\right)$ can be written

$$I_2\left(m_T^2\right) = \int \mathrm{d}\tilde{p}\frac{i}{p^2(p^2 - m_T^2)} = -\frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{m_T^2} + 1\right) . \quad (36)$$

Finally, for $k = 4$ we get

$$\Delta S^{(4)}[H] = \frac{i}{4}\mathrm{Tr}(G\hat{I})^4$$
$$= 4\lambda_1^4 \int \mathrm{d}^4x (HH^\dagger)^2 (s^4 I_1\left(m_T^2\right) + 2s^2c^2 I_2\left(m_T^2\right)$$
$$+ c^4 I_2(0)). \quad (37)$$

Here we need to compute $I_2(0)$. This integral is not only ultraviolet-divergent but also infrared-divergent. Thus, we need to introduce a new infrared cut-off m (obviously the natural value for this cut-off is of the order of v, i.e. the scale of electroweak symmetry breaking). Then we find

$$I_2(0) = \int \mathrm{d}\tilde{p}\frac{i}{p^4} = -\frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{m^2}\right) . \quad (38)$$

Therefore, by using the previous results, it is possible to write the effective Higgs potential parameters as[1]

$$\mu^2 = N_c \frac{m_T^2\lambda_t^2}{4\pi^2}\log\left(1 + \frac{\Lambda^2}{m_T^2}\right) , \quad (39)$$

and

$$\lambda = \frac{N_c}{(4\pi)^2}\left[2\left(\lambda_t^2 + \lambda_T^2\right)\frac{\Lambda^2}{f^2} - \log\left(1 + \frac{\Lambda^2}{m_T^2}\right)\right.$$
$$\times \left(-\frac{2m_T^2}{f^2}\left(\frac{5}{3}\lambda_t^2 + \lambda_T^2\right) + 4\lambda_t^4 + 4\left(\lambda_T^2 + \lambda_t^2\right)^2\right)$$
$$\left. - 4\lambda_T^2\frac{1}{1 + \frac{m_T^2}{\Lambda^2}}\left(\frac{m_T^2}{f^2} - 2\lambda_t^2 - \lambda_T^2\right) - 4\lambda_t^4\log\left(\frac{\Lambda^2}{m^2}\right)\right] , \quad (40)$$

where N_c is the number of colors and λ_t and λ_T are, respectively, the SM top Yukawa coupling and the heavy top Yukawa coupling, given by[2]

$$\lambda_t = \frac{\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$
$$\lambda_T = \frac{\lambda_1^2}{\sqrt{\lambda_1^2 + \lambda_2^2}} . \quad (41)$$

There are several comments on this result worthwhile stressing. First, the effective potential depends on H through the combination $|H|^2 = HH^\dagger$, thus reflecting the fact that the radiative corrections considered preserve the SM $SU(2)_L \times U(1)_Y$ symmetry. However, $\mu^2 > 0$ and $\lambda > 0$, which are the right signs for these corrections to spontaneously break this symmetry down to $U(1)_{em}$. Thus the minima of the effective potential occur whenever $|H|^2 = v^2/2 \equiv \mu^2/(2\lambda)$. By choosing the new vacuum as the state $H = (0, v)/\sqrt{2}$, we recover the above-mentioned SM symmetry breaking. In particular we find that the physical Higgs boson mass is given by $m_H^2 = 2\lambda v^2$. Notice that for $\lambda < 0$ the model would be inconsistent, and that for $\mu^2 < 0$ there would not be spontaneous symmetry breaking.

In spite of having found apparently the same results as in the SM, the LH model has, however, a number of nice properties. First of all, as we have shown, the Higgs potential parameters can be computed in terms of other, more fundamental, parameters. Therefore there is no need to introduce the parameters ad hoc in order to get the

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[1] Our results agree with previous ones for μ^2 (see, for example, [2] and references therein).

[2] Here we assume that $m_t = \lambda_t v$.

appropriate symmetry breaking as it happens in the SM. Symmetry breaking appears as a result of the dynamics, through third-generation quarks radiative corrections, and not as a tree-level consequence of the SM Lagrangian. On the other hand, the Higgs mass can be written as $m_H^2 = 2\mu^2$ and μ^2 is only a logarithmic-divergent quantity at the one-loop level. Therefore, the Higgs mass is not only light, as precision tests of the SM seems to suggest, but also it is free from the undesired quadratic divergences that appear in the original formulation of the SM at one-loop level. Notice also that the quadratic divergences appearing in λ do not alter this result.

Once spontaneous breaking of the SM symmetry has taken place, the Yukawa Lagrangian given above gives rise to a new mass matrix for the t and T quarks (the b quark remains massless). This mass matrix can be diagonalized through a rotation of the left chiral states t_L and T_L given by the angle θ_L, which, for $v \ll f$, can be written

$$\sin\theta_L \simeq \left(\frac{\lambda_1^2}{\lambda_1^2 + \lambda_2^2}\right)\frac{v}{f}, \qquad (42)$$

and by another rotation of the right states t_R and T_R given by the angle θ_R,

$$\sin\theta_R \simeq s\left[1 - c^2\left(\frac{1}{2} - \left(\frac{\lambda_1^2}{\lambda_1^2 + \lambda_2^2}\right)\right)\frac{v^2}{f^2}\right]. \qquad (43)$$

After these rotations the new mass eigenvalues become

$$m_t = \lambda_t v\left[1 + \left(-\frac{1}{3} + \frac{1}{2}\frac{\lambda_1^2}{\lambda_1^2 + \lambda_2^2}\left(1 - \frac{\lambda_1^2}{\lambda_1^2 + \lambda_2^2}\right)\right)\frac{v^2}{f^2}\right] \qquad (44)$$

and

$$m_T = \frac{\lambda_t^2 + \lambda_T^2}{\lambda_T}f - \frac{1}{2}\frac{\lambda_1^2}{\sqrt{\lambda_1^2 + \lambda_2^2}}\left(1 - \frac{\lambda_1^2}{\lambda_1^2 + \lambda_2^2}\right)\frac{v^2}{f}. \qquad (45)$$

4 Constraints on the LH parameter space

Whatever model one considers as a candidate for physics beyond the SM, consistency with the present experimental data is a key requirement. It is well known that indirect constraints from precision electroweak measurements on new physics at the TeV scale are severe. There exist several studies of the corrections to electroweak precision observables in little Higgs models, exploring whether there are regions of the parameter space in which the model is consistent with the data [1, 2, 15, 25–40].

The effective Higgs potential parameters are related with the vacuum expectation value v through $\mu^2 = \lambda v^2$. By imposing this condition, we could extract crucial information on the allowed region of the parameter space in the LH model. To show this, we will consider here the case of the heavy quark contribution to the Higgs potential. In spite of the fact that other contributions, coming from the gauge

bosons, scalars and other higher loops, are relevant, the fermionic sector provides a good illustration of the kind of constraints that it will be possible to set on the LH parameter space by means of the complete effective Higgs potential.

The contributions from the fermion sector of the model to μ^2 and λ are summarized in (39) and (40), respectively. The relevant free parameters of the model are the heavy top mass m_T, the coupling constant λ_T, the symmetry breaking scale f, and the scale Λ. However, there are several relations between them which are worth remarking on. First of all, at lowest order we have

$$m_T = f\frac{\lambda_t^2 + \lambda_T^2}{\lambda_T}, \qquad (46)$$

which in principle, could be tested by LHC experiments [2, 16]. This relation is crucial for the cancelation of quadratic divergence contributions to the Higgs boson mass. Besides, considering m_T to be much larger than 2 TeV would imply a large amount of fine-tuning in the Higgs potential, and thus the heavy top should be below about 2 TeV ($m_T \lesssim 2.5$ TeV) [11, 16]. From the top mass it is also possible to set bounds on the couplings: $\lambda_1, \lambda_2 \geq m_t/v$ or $\lambda_1 \lambda_2 \geq 2(m_t/v)^2$ [15]. As a consequence, we get the bound $\lambda_T \gtrsim 0.5$, which has been considered in our analysis. For the purpose of illustration the dependence of m_T on λ_T and f is shown in Fig. 2a for $0.5 < \lambda_T < 2.5$ and 0.8 TeV $< f < 2$ TeV. Clearly m_T grows linearly with f. Then the condition $m_T \lesssim 2$ TeV implies $f \lesssim 1$ TeV. For $\lambda_T > 1.7$ we get values of the heavy top mass above 2 TeV for $f = 1$ TeV (Fig. 2b). We note that once spontaneous symmetry breaking of the SM is produced, the heavy top mass is reduced by $\mathcal{O}(v^2/f^2)$ terms (see (45)), but the reduction is only about 0.01 TeV. Therefore the previous discussion does not change in a significant way. On the other hand, Λ is expected to be of the order of $\Lambda \sim 4\pi f$ [12–14] and, in addition, electroweak precision tests seem to indicate an experimental lower bound of $\Lambda \gtrsim 10$ TeV [41, 42].

Taking into account the above restrictions on the three parameters λ_T, f and Λ, we now illustrate the kind of constraints that could be set on the LH parameters from the condition $\mu^2 = \lambda v^2$. For the numerical analysis we consider the following ranges: $0.5 < \lambda_T < 2$, 10 TeV $< \Lambda < 12$ TeV and, accordingly, 0.8 TeV $< f < 1$ TeV. Let us first describe the behavior of μ^2 versus λ_T, f and Λ. In general μ^2 corrections increase with f for $0.5 < \lambda_T < 2$, having a minimum for a certain value of λ_T, which corresponds to a minimum of m_T. These corrections also increase with Λ, but less dramatically. We find that the lowest value of μ is $\mu = 0.48$ TeV, corresponding to $\lambda_T = 0.72$, $f = 0.8$ TeV and $\Lambda = 10$ TeV.

Once the corrections to the quartic coupling λ have been computed (see (40)), the consistency of the LH model is constrained by the non-trivial condition (3). The leading contributions of t, b and T to λ in (40) had not been given so far. In the following our purpose will be to use them to illustrate the kind of constraints on the LH model that can be set from the relation between λ and μ^2.
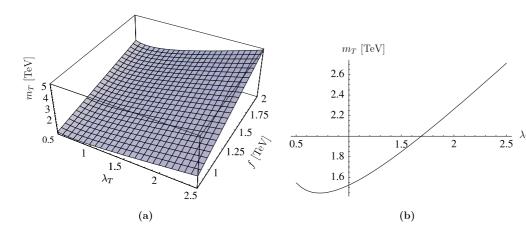
Fig. 2. (a) m_T as a function of λ_T and f, with $0.5 < \lambda_T < 2.5$ and $0.5 < f < 2$ and (b) m_T as a function of λ_T for $f = 1$ TeV

In general, the corrections to λ grow with the scale Λ, but, conversely, we also find that they decrease with the symmetry breaking scale f. Figure 3 shows the surface of solutions for this non-trivial condition. Clearly the allowed region of the parameter space is considerably reduced. The lowest μ value satisfying (3) is $\mu = 0.52$ TeV, when $f = 0.85$ TeV, $\lambda_T = 0.52$ and $\Lambda = 10$ TeV.

However, it is well known that μ is forced by the data to be at most of the order of 200 GeV. Therefore, other contributions must be included in order to obtain the expected μ value. Notice that the Higgs mass parameter in the LH model also receives contributions (not included here) from the vector boson sector (one-loop correction) and from the scalar sector (two-loop correction), which are opposite in sign to the fermion contribution computed here.

Concerning the gauge bosons interactions, we may consider the two different models described in Sect. 2: the original LH with two U(1) groups (*model I*) and the other one with just one U(1) group (*model II*). The contributions from the vector bosons to the mass term of the Higgs field in each case are given, respectively, at one-loop level,
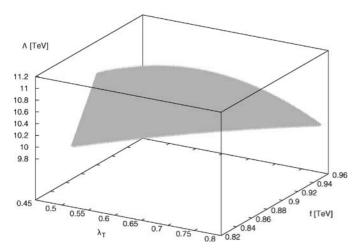
by

$$\mu_{\mathrm{g}}^{2\,\mathrm{I}} = -\frac{3}{64\pi^2}\left(3g^2 M_{W'}^2 \log\left(\frac{\Lambda^2}{M_{W'}^2}+1\right)\right.$$
$$\left. + g'^2 M_{B'}^2 \log\left(\frac{\Lambda^2}{M_{B'}^2}+1\right)\right), \qquad (47)$$

$$\mu_{\mathrm{g}}^{2\,\mathrm{II}} = -\frac{3}{64\pi}\left(3g^2 M_{W'}^2 \log\left(\frac{\Lambda^2}{M_{W'}^2}+1\right) + g'^2 \Lambda^2\right), \qquad (48)$$

where $M_{W'}$ and $M_{B'}$ are the heavy gauge boson masses,

$$M_{W'} = \frac{fg}{2\cos\psi\sin\psi} \quad \text{and} \quad M_{B'} = \frac{fg'}{\sqrt{20}\cos\psi'\sin\psi'}, \qquad (49)$$

with ψ and ψ' being the mixing angles for the W' and B' states. Note that the different results for these two models come from the fact that there is no B' in *model II*.

Let us now study how these vector boson contributions could help to keep μ of order 200 GeV. In order to avoid the gauge masses being too small or too big, we impose $0.1 < \cos\psi < 0.9$ and $0.1 < \cos\psi' < 0.4$. For *model I* we find that the minimum value for μ equals 0.338 TeV for $\lambda_T \simeq 0.7$, $f \simeq 0.8$ TeV, $\Lambda \simeq 10$ TeV, $\cos\psi \simeq 0.1$ and $\cos\psi' \simeq 0.1$. In the case of *model II* we obtain a minimum for μ of 0.2 TeV at $\lambda_T \simeq 0.65$, $f \simeq 0.8$ TeV, $\Lambda \simeq 11.9$ TeV, and $\cos\psi \simeq 0.1$. Therefore, we find that in *model II* it is more natural to find a light Higgs boson than in *model I*. Note, however, that two-loop scalar corrections may also play an important role.

5 Conclusions

In this work we have computed and analyzed the fermion contributions to the low energy effective Higgs potential, and we have illustrated the kind of constraints on the possible values of the LH parameters that can be set by requiring the complete LH effective Higgs potential to reproduce exactly the SM potential.

Fig. 3. Values of Λ, f and λ_T, with $0.5 < \lambda_T < 2$, 0.8 TeV $< f < 1$ TeV and 10 TeV $< \Lambda < 12$ TeV, that satisfy the condition $\mu^2 = \lambda v^2$

We have explored the region of the λ_T, f and Λ parameter space compatible with the condition $\mu^2 = \lambda v^2$, taking $m_T \leq 2\,\text{TeV}$ and $\Lambda \sim 4\pi f$. For this purpose we have computed the leading fermion contributions to the quartic coupling λ at the one-loop level. Since the values obtained for μ are relatively high, the inclusion of the gauge and the scalar sector of the model is needed to reduce μ to its expected value. Therefore, more detailed computations, including the full one-loop gauge boson and the relevant two-loop Goldstone boson contribution, are needed in order to establish the validity of the LH model and its compatibility with the present phenomenological constraint, including the precise form of the Higgs potential. Work is in progress in this direction [43].

References

1. M. Schmaltz, D. Tucker-Smith, Ann. Rev. Nucl. Part. Sci. **55**, 229 (2005) [hep-ph/0502182]
2. M. Perelstein, Prog. Part. Nucl. Phys. **58**, 247 (2007) [hep-ph/0512128]
3. H. Georgi, A. Pais, Phys. Rev. D **10**, 539 (1974)
4. H. Georgi, A. Pais, Phys. Rev. D **12**, 508 (1975)
5. M.J. Dugan, H. Georgi, D.B. Kaplan, Nucl. Phys. B **254**, 299 (1985)
6. H. Georgi, D.B. Kaplan, Phys. Lett. B **145**, 216 (1984)
7. H. Georgi, D.B. Kaplan, Phys. Lett. B **136**, 183 (1984)
8. H. Georgi, D.B. Kaplan, P. Galison, Phys. Lett. B **143**, 152 (1984)
9. D.B. Kaplan, H. Georgi, S. Dimopoulos, Phys. Lett. B **136**, 187 (1984)
10. S. Dimopoulos, J. Preskill, Nucl. Phys. B **199**, 206 (1982)
11. N. Arkani-Hamed, A.G. Cohen, E. Katz, A.E. Nelson, JHEP **0207**, 034 (2002) [hep-ph/0206021]
12. A. Manohar, H. Georgi, Nucl. Phys. B **234**, 189 (1984)
13. M.A. Luty, Phys. Rev. D **57**, 1531 (1998) [hep-ph/9706235]
14. A.G. Cohen, D.B. Kaplan, A.E. Nelson, Phys. Lett. B **412**, 301 (1997) [hep-ph/9706275]
15. T. Han, H.E. Logan, B. McElrath, L.T. Wang, Phys. Rev. D **67**, 095004 (2003) [hep-ph/0301040]
16. M. Perelstein, M. Peskin, A. Pierce, Phys. Rev. D **69**, 075002 (2004) [hep-ph/0310039]
17. I. Low, W. Skiba, D. Smith, Phys. Rev. D **66**, 072001 (2002) [hep-ph/0207243]
18. D.E. Kaplan, M. Schmaltz, JHEP **0310**, 039 (2003) [hep-ph/0302049]
19. S. Chang, J.G. Wacker, Phys. Rev. D **69**, 035002 (2004) [hep-ph/0303001]
20. W. Skiba, J. Terning, Phys. Rev. **68**, 075001 (2003) [hep-ph/0305302]
21. S. Chang, JHEP **0312**, 057 (2003) [hep-ph/0306034]
22. J.A. Casas, J.R. Espinosa, I. Hidalgo, JHEP **0503**, 038 (2005) [hep-ph/0502066]
23. F. Bazzocchi, M. Fabbrichesi, M. Piai, Phys. Rev. D **72**, 095019 (2005) [hep-ph/0506175]
24. A. Dobado, A. Gómez-Nicola, A.L. Maroto, J.R. Peláez, Effective Lagrangians for the Standard Model (Springer, Heidelberg, 1997)
25. Z. Han, W. Skiba, Phys. Rev. D **72**, 035005 (2005) [hep-ph/0506206]
26. H.E. Logan, Phys. Rev. D **70**, 115003 (2004) [hep-ph/0405072]
27. T. Han, H.E. Logan, B. McElrath, L.T. Wang, Phys. Lett. B **563**, 191 (2003)
28. T. Han, H.E. Logan, B. McElrath, L.T. Wang, Phys. Lett. B **B603**, 257 (2004) [hep-ph/0302188] [Erratum]
29. C. Csaki, J. Hubisz, G.D. Kribs, P. Meade, J. Terning, Phys. Rev. D **67**, 115002 (2003) [hep-ph/0211124]
30. C. Csaki, J. Hubisz, G.D. Kribs, P. Meade, J. Terning, Phys. Rev. D **68**, 035009 (2003) [hep-ph/0303236]
31. J.L. Hewett, F.J. Petriello, T.G. Rizzo, JHEP **0310**, 062 (2003) [hep-ph/0211218]
32. T. Gregoire, D.R. Smith, J.G. Wacker, Phys. Rev. D **69**, 115008 (2004) [hep-ph/0305275]
33. M.C. Chen, S. Dawson, Phys. Rev. D **70**, 015003 (2004) [hep-ph/0311032]
34. W. Kilian, J. Reuter, Phys. Rev. D **70**, 015004 (2004) [hep-ph/0311095]
35. G. Marandella, C. Schappacher, A. Strumia, Phys. Rev. D **72**, 035014 (2005) [hep-ph/0502096]
36. M.C. Chen, Mod. Phys. Lett. A **21**, 621 (2006) [hep-ph/0601126]
37. S.R. Choudhury, A.S. Cornell, N. Gaur, A. Goyal, hep-ph/0604162
38. J.A. Conley, J. Hewett, M.P. Le, Phys. Rev. D **72**, 115014 (2005) [hep-ph/0507198]
39. C.O. Dib, R. Rosenfeld, A. Zerwekh, AIP Conf. Proc. **815**, 296 (2006) [hep-ph/0509013]
40. Z. Berezhiani, P.H. Chankowski, A. Falkowski, S. Pokorski, Phys. Rev. Lett. **96**, 031801 (2006) [hep-ph/0509311]
41. R. Barbieri, A. Strumia, hep-ph/0007265
42. R. Barbieri, A. Pomarol, R. Rattazzi, A. Strumia, Nucl. Phys. B **703**, 127 (2004) [hep-ph/0405040]
43. A. Dobado, L. Tabares, S. Peñaranda, hep-ph/0612131